1. Express the statement that is to be proved in the form “for all n ≥ b, P(n)” for a fixed

integer b.

2. Write out the words “Basis Step.” Then show that P(b) is true, taking care that the correct

value of b is used. This completes the first part of the proof.

3. Write out the words “Inductive Step.”

4. State, and clearly identify, the inductive hypothesis, in the form “assume that P(k) is true

for an arbitrary fixed integer k ≥ b.”

5. State what needs to be proved under the assumption that the inductive hypothesis is true.

That is, write out what P(k + 1) says.

6. Prove the statement P(k + 1) making use the assumption P(k). Be sure that your proof

is valid for all integers k with k ≥ b, taking care that the proof works for small values

of k, including k = b.

7. Clearly identify the conclusion of the inductive step, such as by saying “this completes

the inductive step.”

8. After completing the basis step and the inductive step, state the conclusion, namely that

by mathematical induction, P(n) is true for all integers n with n ≥ b.

1. Let P(n) be the statement that Qn has no cut vertices for all n >= 2.
2. Qn is an n-dimensional hypercube graph that has vertices (V) and edges (E) where |V| = 2n (see figure 1 above). It is important to note that each vi that exists in V has a degree of n and n neighbors, because each bit in the string in n can differ by 1.
3. Basis Step: P(2) is true because |V| = 2^n = 4 and each vi that exists in V is connected to two other vi that exist in V. This means deg(vi) = 2 for all vi that exist in V. If one vertex in Q2 is removed this will decrement each of its neighbor’s degrees by 1 creating a graph of three nodes with degrees of 2, 1, 1 that is still connected (see figure 2 above). Without loss of generality, any of the 4 vertices can be removed from Q2 without creating a disconnected subgraph, which means that P(2) is true.
4. Inductive Hypothesis: P(k) is true for all integers k>=2. That is, Qk for all integers k >= 2 do not contain a cut vertex.
5. Inductive Step: Then for Qk+1, |V|=k+1 and each deg(vi)=k+1 for vi that exist in V. By the inductive hypothesis, Qk does not contain a cut vertex meaning deg(vi)-1 for all vi exist in V is greater than zero (because k>=2 then k-1 >= 1). Because Qk+1 increments each vertices’ degree in Qk by 1, a cut vertex would decrement each of missing vertices’ neighbor’s degree by one leaving degree of k. Since k+1-1>= 0 we know the remaining graph will still be connected. Without loss of generality we could remove any vi from Qk+1 and still have a connected graph.
6. By mathematical induction, P(n) is true for all integers n with n>=2.

Because the inductive hypothesis a cut vertex whenever the inductive hypothesis is true. Since Qk+1 adds a neighbor to each of the vertices in Qk. By the inductive hypothesis Qk has no cut vertices. then the deg(vi) for all vi exist in V is equal to k+1. If one vertex is removed, then each neighbor of the missing vertex will now have